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We consider the Grand Unified SU(5) model with a small or vanishing cubic term in the adjoint scalar field in the potential. This gives the model an approximate or exact Z_2 symmetry whose breaking leads to domain walls. The simplest domain wall has the structure of a kink across which the Higgs field changes sign ($\Phi \to -\Phi$) and inside which the full SU(5) is restored. The kink is shown to be perturbatively unstable for all parameters. We then construct a domain wall solution that is lighter than the kink and show it to be perturbatively stable for a range of parameters. The symmetry in the core of this domain wall is smaller than that outside. The interactions of the domain wall with magnetic monopole is discussed and it is shown that magnetic monopoles with certain internal space orientations relative to the wall pass through the domain wall. Magnetic monopoles in other relative internal space orientations are likely to be swept away on collision with the domain walls, suggesting a scenario where the domain walls might act like optical polarization filters, allowing certain monopole "polarizations" to pass through but not others. As SU(5) domain walls will also be formed at small values of the cubic coupling, this leads to a very complicated picture of the evolution of defects after the Grand Unified phase transition.

I. INTRODUCTION

Topological defects can be produced at a symmetry breaking phase transition and would be long-lived relics of the symmetric phase. If topological defects were produced during a phase transition in the very early universe, they could survive until the present epoch and thus provide a window to the very early universe. The lack of observable defects in the present universe helps place strong constraints on particle physics model building and early universe cosmology.

A prototype symmetry breaking relevant for Grand Unified particle physics is

$$\mathrm{SU}(5) \to [\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)]/\mathrm{Z}_6 \ .$$

The corresponding phase transition would produce magnetic monopoles. If the only factors affecting the evolution of the monopoles are the sub-luminal expansion of the universe and monopole-antimonopole Coulombic interactions, the monopole abundance grossly violates the observed absence of monopoles in the present universe. The monopole over-abundance problem is solved by invoking superluminal universal expansion (*i.e.* inflation [1]) or by extending the particle physics model so that the U(1) symmetry gets broken for a short duration leading to confining forces between monopoles and antimonopoles [2] and thus enhancing their annihilation rate*. Recently [4,5] we have investigated the possibility

*There is another possibility - that the Grand Unified that the Grand Unified phase transition may also have produced a network of domain walls together with the magnetic monopoles. These walls would interact with the monopoles and sweep them away, reducing their abundance to an acceptably low level. It is to pursue this scenario in greater detail that we now study the structure of domain walls in the SU(5) model.

The bosonic sector of the SU(5) model is:

$$L = \mathrm{Tr}(D_\mu \Phi)^2 - V(\Phi) \tag{1}$$

where Φ is an adjoint of SU(5), $D_\mu \Phi = \partial_\mu \Phi - ig[X_\mu, \Phi]$ X_μ are the gauge fields, and the potential is given by:

$$V(\Phi) = -m^2\,\mathrm{Tr}(\Phi^2) + h[\mathrm{Tr}(\Phi^2)]^2 + \lambda\,\mathrm{Tr}(\Phi^4) + \gamma\,\mathrm{Tr}(\Phi^3) - V_0\ , \tag{2}$$

where, V_0 is a constant that we will choose below. The SU(5) symmetry is broken to $[SU(3)\times SU(2)\times U(1)]/Z_6$ if the Higgs acquires a vacuum expectation value (VEV) equal to

$$\Phi_0 = \frac{\eta}{2\sqrt{15}}\mathrm{diag}(2,2,2,-3,-3)\ , \tag{3}$$

where

$$\eta = \frac{m}{\sqrt{\lambda'}}\ , \tag{4}$$

$$\lambda' \equiv h + \frac{7}{30}\lambda\ . \tag{5}$$

For the potential to have its global minimum at $\Phi = \Phi_0$, the parameters are constrained to satisfy:

$$\lambda \geq 0\ , \quad \lambda' \geq 0\ . \tag{6}$$

For the global minimum to have $V(\Phi_0) = 0$, in eq. (2) we set

$$V_0 = -\frac{\lambda'}{4}\eta^4\ . \tag{7}$$

The model in eq. (1) does not have any topological domain walls because the vacua related by $\Phi \to -\Phi$ are not degenerate. However if γ is small, there are walls connecting the two kinds of vacua that are almost topological. In our analysis we will set $\gamma = 0$, in which case the symmetry of the model is SU(5)×Z_2 and an expectation of Φ breaks the Z_2 symmetry leading to topological domain walls in addition to the magnetic monopoles arising from the SU(5) breaking.

In this paper we will study the domain walls present in the SU(5)×Z_2 model. The simplest kind of domain

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phase transition never occurred and hence there never was a monopole over-abundance problem [3].

wall is the kink that has been studied in a single scalar field model with $Z_2 \to 1$ (*eg.* [6]). In [5] we studied the interaction of the SU(5) kink with magnetic monopoles and found that the monopoles spread out along the kink on collision and never pass through. This confirmed the conjecture in Ref. [4] that kinks could sweep away magnetic monopoles. However, the investigations of this paper show that the kink solution of the $SU(5)\times Z_2$ model is unstable to perturbations. The model contains another domain wall solution that is lighter than the kink and is perturbatively stable. The adjoint field does not vanish in the core of these new domain wall solutions and hence only a subgroup of the SU(5) is restored at the center. For this reason, the interactions of these domain walls with magnetic monopoles is expected to be much more complex (as compared to the kink), depending on the particular group orientation of the monopole relative to the wall.

We will begin our analysis in Sec. II by constructing the kink and performing the stability analysis. Then in Sec. III we will proceed to construct the domain wall in the model, prove that it is lighter than the kink, and show that it is perturbatively stable for a range of parameters. In Sec. IV we will consider the interaction of monopoles and domain walls and show that a monopole whose orientation in group space is aligned with a colliding domain wall, will pass through and not get swept away. We further conjecture that monopoles that are misaligned with the domain wall will be swept away but have not yet shown this. We draw an analogy of the sweeping out process with that of a polarization filter that "sweeps out" orthogonally polarized light and only lets through a certain polarization.

II. SU(5) KINK: SOLUTION AND STABILITY

The kink solution is the Z_2 kink along the Φ_0 direction (see eq. (3)). Therefore:

$$\Phi_k = \tanh(\sigma z)\Phi_0 \tag{8}$$

with $\sigma \equiv m/\sqrt{2}$ (see eq. (5)), and all the gauge fields vanish. It is straightforward to check that Φ_k solves the equations of motion with the boundary conditions $\Phi(z = \pm\infty) = \pm\Phi_0$.

As is well-known [6], the mass (per unit area) of the kink is:

$$M_k = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda'} \ . \tag{9}$$

Here we will examine the stability of the kink under general perturbations. So we write:

$$\Phi = \Phi_k + \Psi \tag{10}$$

Since the kink solution is invariant under translations and rotations in the $xy-$plane, it is easy to show that the perturbations that might cause an instability arise from perturbations of the scalar field and can only depend on z. Therefore we may set the gauge fields to zero and take $\Psi = \Psi(t, z)$.

The Z_2 kink is stable and hence we can restrict the scalar perturbations to be orthogonal to Φ_k. Furthermore, since the stability of the kink to diagonal perturbations has already been studied in Ref. [4], we only have to consider perturbations that cannot be diagonalized by a global SU(3)×SU(2)×U(1) transformation that leaves the kink invariant. Therefore we can write:

$$\Psi = \sum_{i=1}^{12} \psi^i T^i \, , \tag{11}$$

where T^i are all generators of SU(5) that do not commute with Φ_0.

Next we analyze the linearized Schrodinger equation for small excitations $\psi^i = \psi_0^i(z) exp(-i\omega t)$ in the background of the kink:

$$[-\partial_z^2 - m^2 + \phi_k^2(z)(h + \lambda r_i)]\psi_0^i = \omega_i^2 \psi_0^i \ \ , \tag{12}$$

where $\phi_k \equiv \tanh(\sigma z)$ and $r_i = 7/30$. Since this equation is identical for excitations along any of the 12 directions, we can drop the index i. The kink is unstable if there is a solution to eq. (12) with a negative ω^2. Substituting eq. (8) into eq. (12) yields:

$$[-\partial_z^2 + m^2(\tanh^2(\sigma z) - 1)]\psi_0 = \omega^2 \psi_0 \ \ . \tag{13}$$

This equation has a bound state solution $\psi_0 \propto \mathrm{sech}(\sigma z)$ with the eigenvalue $\omega^2 = -m^2/2$. Since this result is independent of the parameters in the potential, we conclude that the kink in SU(5) is always unstable.

III. DOMAIN WALL

The domain wall solution is obtained if we choose the gauge fields to vanish at infinity and the scalar field to satisfy the boundary conditions:

$$\begin{aligned}\Phi(z = -\infty) = \Phi^- &\equiv \frac{\eta}{2\sqrt{15}} \mathrm{diag}(2, -3, 2, 2, -3) \\ &= \eta \sqrt{\frac{5}{12}} (\lambda_3 + \tau_3) + \frac{\eta}{6} (Y - \sqrt{5}\lambda_8)\end{aligned} \tag{14}$$

and

$$\begin{aligned}\Phi(z = +\infty) = \Phi^+ &\equiv \frac{\eta}{2\sqrt{15}} \mathrm{diag}(3, -2, -2, 3, -2) \\ &= \eta \sqrt{\frac{5}{12}} (\lambda_3 + \tau_3) - \frac{\eta}{6} (Y - \sqrt{5}\lambda_8) \ .\end{aligned} \tag{15}$$

Here λ_3, λ_8, τ_3 and Y are the diagonal generators of SU(5):

$$\lambda_3 = \frac{1}{2}\mathrm{diag}(1,-1,0,0,0) \ , \tag{16}$$

$$\lambda_8 = \frac{1}{2\sqrt{3}}\mathrm{diag}(1,1,-2,0,0) \ , \tag{17}$$

$$\tau_3 = \frac{1}{2}\mathrm{diag}(0,0,0,1,-1) \ , \tag{18}$$

$$Y = \frac{1}{2\sqrt{15}}\mathrm{diag}(2,2,2,-3,-3) \ . \tag{19}$$

Note that a local SU(5) transformations can be used to rotate Φ^+ into $-\Phi^-$ so that the boundary conditions are like those of the kink with $\Phi(z=+\infty) = -\Phi(z=-\infty)$. However, then the solution for the domain wall will not be diagonal at all z. We prefer to use the above boundary conditions so that the solution is diagonal throughout.

The domain wall solution can be written as

$$\Phi_{DW}(z) = a(z)\lambda_3 + b(z)\lambda_8 + c(z)\tau_3 + d(z)Y \ . \tag{20}$$

The functions a, b, c, and d must satisfy the static equations of motion:

$$a'' = [-m^2 + (h + \frac{2\lambda}{5})d^2 + (h + \frac{\lambda}{2})(a^2+b^2) + hc^2]a + \frac{2\lambda abd}{\sqrt{5}} \tag{21}$$

$$b'' = [-m^2 + (h + \frac{2\lambda}{5})d^2 + (h + \frac{\lambda}{2})(a^2+b^2) + hc^2]b + \frac{\lambda d}{\sqrt{5}}(a^2 - b^2) \tag{22}$$

$$c'' = [-m^2 + (h + \frac{9\lambda}{10})d^2 + (h + \frac{\lambda}{2})c^2 + h(a^2+b^2)]c \tag{23}$$

$$d'' = [-m^2 + (h + \frac{7\lambda}{30})d^2 + (h + \frac{2\lambda}{5})(a^2+b^2) + (h + \frac{9\lambda}{10})c^2]d + \frac{\lambda b}{\sqrt{5}}(a^2 - \frac{b^2}{3}) \ , \tag{24}$$

where primes refer to derivatives with respect to z. For reference, the kink solution (eq. (8)) corresponds to $a(z) = 0 = b(z) = c(z)$ and $d(z) = \eta\tanh(\sigma z)$.

The equations of motion for b and c and can be solved quite easily:

$$b(z) = -\sqrt{5}d(z) \ , \quad c(z) = a(z) \ . \tag{25}$$

This is consistent with the boundary conditions in eqs. (14) and (15). In addition, we require

$$a(z=\pm\infty) = +\eta\sqrt{\frac{5}{12}} \ , \quad d(z=\pm\infty) = \mp\frac{\eta}{6} \ . \tag{26}$$

Then the remaining equations we need to solve are:

$$a'' = \left[-m^2 + \left(6h + \frac{9}{10}\lambda\right)d^2 + \left(2h + \frac{\lambda}{2}\right)a^2\right]a \tag{27}$$

$$d'' = \left[-m^2 + \left(6h + \frac{39}{10}\lambda\right)d^2 + \left(2h + \frac{3\lambda}{10}\right)a^2\right]d \tag{28}$$

These equations can be written in a cleaner form by rescaling:

$$A(z) = \sqrt{\frac{12}{5}}\frac{a}{\eta} \ , \quad D(z) = -6\frac{d}{\eta} \ , \quad Z = mz \ . \tag{29}$$

Then

$$A'' = \left[-1 + \frac{(1-p)}{5}D^2 + \frac{(4+p)}{5}A^2\right]A \tag{30}$$

$$D'' = \left[-1 + pD^2 + (1-p)A^2\right]D \tag{31}$$

where primes on A and D denote differentiation with respect to Z, and

$$p = \frac{1}{6}\left[1 + \frac{5\lambda}{12\lambda'}\right] \ . \tag{32}$$

Note that $p \in [1/6, \infty)$ because of the constraints in eq. (6). The boundary conditions now are:

$$A(z=\pm\infty) = +1 \ , \quad D(z=\pm\infty) = \pm 1 \ . \tag{33}$$

This system of equations has been solved by numerical relaxation and a sample solution is shown in Fig. 1. To find an approximate analytical solution, assume that $|A''/A| << 1$ is small everywhere. This assumption will be true for a certain range of the parameter p which we can later determine. Then the square bracket on the right-hand side of eq. (30) is very small. This gives:

$$A \simeq \left[\frac{5}{4+p}\left\{1 - \frac{(1-p)}{5}D^2\right\}\right]^{1/2} \tag{34}$$

We insert this expression for A in eq. (31) and obtain the kink-type differential equation:

$$D'' = q[-1 + D^2]D \ , \tag{35}$$

where

$$q = \frac{6p-1}{p+4} = \frac{6\lambda}{\lambda + 60\lambda'} \tag{36}$$

and the solution is:

$$D(Z) \simeq \tanh\left(\sqrt{\frac{q}{2}}Z\right) \tag{37}$$

The parameter q lies in the interval $[0, 6]$. For $q = 1$ (*i.e.* $p = 1$) it is easy to check that this analytical solution is exact.

We can now check that our assumption $|A''/A| << 1$ is self-consistent provided p is not much larger than a few.

The energy density for the fields A and D can be found from the Lagrangian in eq. (1) together with the ansatz in eq. (20), the solution for b and c in eq. (25) and the rescalings in eq. (29). The resulting expression for the energy per unit area of the domain wall is:

$$M_{DW} = \frac{m^3}{12\lambda'}\int dZ[5A'^2 + D'^2 + V(A, D)] \tag{38}$$

where,

$$V(A, D) = -5A^2 - D^2 + \frac{(p+4)}{2}A^4 + \frac{p}{2}D^4 + (1-p)A^2D^2 + 3\ . \tag{39}$$

The energy can be found numerically. However, here we will find an approximate analytic result. We can insert the approximate solution given above in eq. (38) but this leads to an expression that is not transparent. Instead it is more useful to consider another approximation for A and D:

$$A \simeq 1\ , \quad D \simeq \tanh\left(\sqrt{\frac{p}{2}}Z\right)\ . \tag{40}$$

(This approximation is exact for $p = 1$.) A straightforward evaluation then gives:

$$M_{DWapprox} = M_k\frac{\sqrt{p}}{6} \tag{41}$$

where, M_k is given in eq. (9).

We can now compare the domain wall energy to the kink energy. If the domain wall is the least energy solution for the given boundary conditions, the energy of the exact solution for the domain wall will be bounded above by the energy of the approximate solution. Note that this will be true even if the approximation used to find the analytical solution is not good. Hence this simple argument shows that the domain wall is lighter than the kink for $p < 36$, or for $h/\lambda > -6.94/30$. A numerical analysis shows that the domain wall is lighter than the kink even in the range $-6.94/30 \geq h/\lambda > -7/30$. Therefore the domain wall is lighter than the kink for the full range of parameters specified in eq. (6).

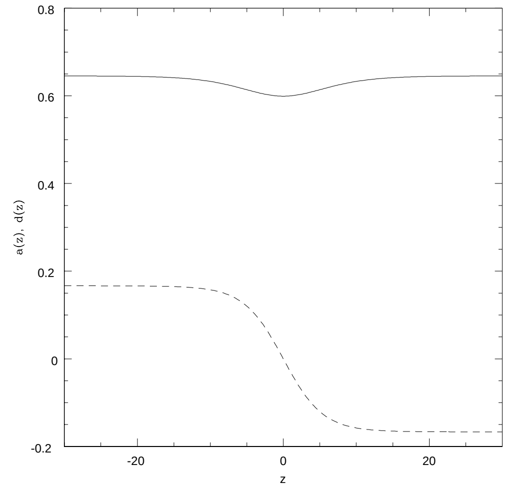

FIG. 1. The domain wall solution for $\lambda = 1$ and $h = -0.2$ ($p = 2.25$). The solid line shows $a(z)$ and the dashed line shows $d(z)$.

Next we study the stability of the domain wall solution. It is easy to show that the solution is stable to diagonal perturbations, so here we focus on off-diagonal perturbations. We write:

$$\Phi = \Phi_{DW}(z) + \sum_{a=1}^{20} \psi^a(z) N^a \ , \qquad (42)$$

where N^a are the non-diagonal generators of $SU(5)$ and ψ^a are small perturbations satisfying the boundary conditions $\psi^a(\pm\infty) = 0$. Let us first consider the contribution to the energy density due to fields ψ^a. To second order in perturbations the contributions from different modes, labeled by index a, do not couple. A more detailed analysis shows that the mode corresponding to

$$N_1 \equiv \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (43)$$

is one of the 8 modes that are most unstable. Let ψ be any of these 8 fields. The contribution to the energy density due to ψ is:

$$E_\psi = \frac{1}{2}(\psi')^2 - \frac{m^2}{2}\psi^2 + \frac{h}{4}(\psi^2 + 2a^2 + 6d^2)^2 + \frac{\lambda}{4}\psi^2(a^2 + \frac{9}{5}d^2) + \text{higher order terms} \ , \qquad (44)$$

where a and d are defined by eq. (20). As in the case of the kink, we are interested in the linearized Schrodinger equation for small excitations $\psi = \psi_0(z)exp(-i\omega t)$ in the background of the diagonal domain wall solution. Eq. (44) leads to

$$[-\partial_z^2 - m^2 + (6h + \frac{9}{10}\lambda)d^2 + (2h + \frac{\lambda}{2})a^2]\psi_0 = \omega^2\psi_0 \ . \tag{45}$$

Comparing this with eq. (27) allows us to write:

$$-\psi_0'' + \frac{a''}{a}\psi_0 = \omega^2\psi_0 \ . \tag{46}$$

If $a''/a = 0$, as happens when $p = 1$, then there are no non-trivial solutions to eq. (46) that satisfy the correct boundary conditions. Therefore, the diagonal domain wall solution is stable for at least one choice of parameters in the potential, namely, for $p = 1$. By continuity there is a range of parameters around $p = 1$ for which the domain wall is perturbatively stable.

IV. INTERACTION WITH MONOPOLES AND DISCUSSION

To understand the interaction of the domain wall with magnetic monopoles, it is first useful to understand the structure of the domain wall core. Since $a(z)$ is non-zero inside the domain wall, $\Phi_{DW}(z = 0) = a(0)(\lambda_3 + \tau_3) \propto \mathrm{diag}(1, -1, 0, 1, -1)$. Therefore, the symmetry inside the core is K $\equiv$ SU(2)×SU(2)×U(1)×U(1). The first SU(2) factor arises due to rotations in the 2×2 block with the entries equal to 1 in Φ_{DW} (first and fourth rows and columns). The second SU(2) factor is due to the block with entries equal to -1 (second and fifth rows and columns). The two U(1) factors arise since there are two diagonal generators of SU(5) aside from those already accounted for in the two SU(2) factors, that commute with $\Phi_{DW}(z = 0)$. (We are ignoring any discrete factors that might be present.) The symmetry group K within the domain wall is to be contrasted with the full SU(5) symmetry which is restored within the kink. The fact that only a subgroup of the SU(5) symmetry is restored in the core of the wall means that the interaction of the monopole will now depend on the particular embedding of the monopole in SU(5) and its orientation in internal space relative to the domain wall.

Consider a magnetic monopole whose winding lies in the fourth and fifth rows and columns of Φ. Staying close to the notation of [5] we write the scalar field of such a monopole as:

$$\Phi_M(r) = P(r)\sum_{a=1}^{3} x^a \tau_a + M(r)\lambda_8' + N(r)Y \ , \tag{47}$$

where $\{\tau_a\}$ are the SU(2) generators (see eq. (18)) and

$$\lambda_8' \equiv \frac{1}{2\sqrt{3}}\mathrm{diag}(1, -2, 1, 0, 0) = \frac{\sqrt{3}}{2}\lambda_3 - \frac{1}{2}\lambda_8.$$

The non-zero gauge fields are:

$$X_i = \sum_{a=1}^{3} X_i^a \tau_a \ ,$$

$$X_i^a = \epsilon^a{}_{ij} \frac{x^j}{er^2} (1 - K(r)) \, . \tag{48}$$

The monopole profile functions, $P(r)$, $M(r)$, $N(r)$ and $K(r)$, are solutions of the static equations of motion with boundary conditions:

$$P(\infty) = \eta \sqrt{\frac{5}{12}} \ , \ M(\infty) = \eta \frac{\sqrt{5}}{3} \ , \tag{49}$$

$$N(\infty) = \frac{\eta}{6} \ , \ K(\infty) = 1. \tag{50}$$

When the monopole and the wall are very far from each other, the combined field configuration can be described by the following ansatze:

$$\begin{aligned} \Phi_{M+DW} &= P(r) \frac{c(z')}{c(-\infty)} \sum_{a=1}^{3} x^a \tau_a + N(r) \frac{d(z')}{d(-\infty)} Y \\ &+ M(r) \left[\frac{\sqrt{3}}{2} \frac{a(z')}{a(-\infty)} \lambda_3 - \frac{1}{2} \frac{b(z')}{b(-\infty)} \lambda_8 \right] \ , \end{aligned} \tag{51}$$

where $z' = z - z_0$ and z_0 is the initial position of the domain wall. When r is small $\Phi_{M+DW} \to \Phi_M$ (eq. (47)) and when z' is small $\Phi_{M+DW} \to \Phi_{DW}$ (eq. (20)) along the z-direction. The gauge fields are the same as for the monopole alone. We have purposely chosen the embedding of the monopole so that all interesting dynamics of the monopole-wall interaction is restricted to the fourth and fifth rows and columns of Φ_{M+DW}. This follows from the equations of motion and the commutation properties of the generators appearing in the ansatze (51). Let us then only consider the relevant part of the Φ_{M+DW} matrix:

$$\begin{aligned} \Phi_{2\times 2} \equiv & \frac{1}{2} P(r) \frac{a(z')}{a(-\infty)} \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix} \\ & - \frac{3}{2\sqrt{15}} N(r) \frac{d(z')}{d(-\infty)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \, . \end{aligned} \tag{52}$$

The form of $\Phi_{2\times 2}$ suggests that the only field that is going to be considerably affected by the domain wall is $N(r)$ because a is roughly constant in space. There is no angular dependence in the term with $N(r)$ in eq. (47) and hence $N(r)$ does not contribute to the winding of the monopole. Therefore, we do not expect the wall to affect the winding. Essentially the reason is that the SU(2) subgroup in which the monopole winding is located is not restored on the wall. We have checked that the monopole passes right through the wall explicitly in this case by numerically colliding the monopole and the wall.

If the magnetic monopole winding lies in the first and fourth block, it will experience a region of restored SU(2) symmetry inside the domain wall and hence we conjecture that such monopoles will unwind on the wall. If our conjecture is correct, the domain walls behave similarly to optical polarization filters, allowing monopoles with certain internal space polarizations to pass through and annihilating other polarizations. The detailed analysis of all possible monopole embeddings is a challenging project, both numerically and analytically, since one cannot avoid dealing with a large number of the fields present in SU(5).

There is another possibility that is worth pointing out. If a domain wall and a magnetic monopole are misaligned in internal space, it may not be possible to superpose the two solutions so as to get a monopole and a domain wall together. (Such a situation is known to occur when attempting to construct multimonopole or multistring solutions.) Then it is likely that there will be a long range force between the domain wall and misaligned monopole that will bring them together. On coming together the monopole could get annihilated on the wall, or else in some cases, it may get aligned and then pass through the wall.

Our considerations point to a very complicated aftermath of the GUT phase transition. Domain walls and magnetic monopoles would both be produced and would start interacting. The outcome of an interaction would depend on the internal space orientations of the monopole relative to the domain wall. Any given domain wall would be transparent to some monopoles but not to others. The relaxation of the system would depend on whether a monopole encounters a sufficient number of randomly oriented (in internal space) domain walls, at least one of which might sweep it away. It remains to be seen if domain walls can provide a means to solve the cosmological monopole over-abundance problem.

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